Finite-band pulse collection and temporal containment in a scalar solar-lens model

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Abstract

A geometrical arrival-time span does not determine the waveform measured by a finite receiver. We calculate a finite-band scalar field for a coherent transmitter, a spherical solar gravitational phase and an absorbing solar disk, then distinguish area-integrated intensity from coherent coupling into one fixed spatial mode. For a one-metre receiver at 650 AU, a source at one parsec, and a 20 fs intensity-FWHM pulse centred at one micrometre, the estimated maximum fraction of area-integrated energy collectable into a mode with 98% temporal containment is 0.064–0.100 for a 30 fs window and 0.845–0.864 for a 50 fs window. These are numerical envelopes, not certified total-error bounds: quadrature and roundoff retain an empirical allowance. A finer discretization and a separate spectral sinc integration check support numerical stability. The calculation is a receiver-dependent benchmark, not a prediction for the real solar environment, a communications-capacity calculation, or a demonstrated instrument. The work is archived with reproducible code; extension to the solar quadrupole and plasma is left unperformed.

1 Question and relation to the preceding calculation

The precursor note [1] establishes an approximately 25.956 fs principal-ray arrival span across a one-metre pupil at 650 AU in a specified simplified geometry. It does not calculate a received pulse, and its ray span is not a pulse duration or a data-rate limit. Its two supplied numerical certificates were reproduced before the present calculation.

This note asks a different question: how much energy can a fixed passive spatial receiver mode collect while placing at least 98% of its own received energy inside a chosen time interval? The distinction matters because an aperture that sums local intensities and an aperture that coherently projects onto a spatial mode need not measure the same temporal waveform.

Scalar diffraction, Fourier synthesis of finite-band fields, spatial-mode projection and quadratic-form optimization are established methods. The contribution here is a specified numerical application combining them, with an opaque disk and a finite coherent source, and a reproducible collection–containment comparison. This note makes no literature-wide priority claim. A monochromatic spatial point-spread function by itself does not specify a temporal impulse response: coherent spectral phase, source illumination and the detector observable must also be supplied.

2 Model and observables

We use $c = 299792458\,\mathrm{m\,s^{-1}}$, $GM_\odot = 1.32712440018 \times 10^{20}\,\mathrm{m^3\,s^{-2}}$, $R = 6.957 \times 10^8\,\mathrm{m}$, $r_s = 2GM_\odot/c^2$, and $\mathrm{AU} = 149597870700\,\mathrm{m}$. The source and receiver distances are $z_s = (648000/\pi)\,\mathrm{AU}$ and $z_r = 650\,\mathrm{AU}$. The receiver is a centred disk of radius $a_r = 0.5\,\mathrm{m}$. The source field has flat phase and frequency-independent spatial profile

$$S(u) = C\exp(-u^2/w_0^2)\,\mathbf{1}_{u\le d}, \quad w_0 = 0.5\,\mathrm{m}, \quad d = 1\,\mathrm{m}, \quad C^{-2} = \frac{\pi w_0^2}{2}(1 - e^{-2d^2/w_0^2}). \tag{1}$$

Thus $\int |S|^2 dA = 1$. The scalar thin-phase model uses a spherical gravitational logarithmic phase and transmission zero for $r < R$. It excludes plasma, nonspherical solar gravity, absorption outside the specified disk, polarization and scattering. Fresnel propagation is paraxial. Within these assumptions, define $k = 2\pi f/c$, $Z = (z_s^{-1} + z_r^{-1})^{-1}$ and

$$F(r) = \frac{r^2}{2Z} - 2r_s \log(r/R), \tag{2}$$

$$I(k,r) = \frac{kC}{iz_s} \int_0^d u\, e^{-u^2/w_0^2} e^{iku^2/(2z_s)} J_0(kru/z_s)\, du, \tag{3}$$

$$H(f,\rho) = \frac{k}{iz_r} e^{ik\rho^2/(2z_r)} \int_R^\infty rI(k,r) J_0(kr\rho/z_r) e^{ik[F(r)-F(b)]}\, dr, \quad b = \sqrt{2r_s Z}. \tag{4}$$

The real radial integral is defined by an Abel limiting factor $e^{-\epsilon r^2}$ as $\epsilon \downarrow 0$. Common propagation and saddle delays have been removed; all quoted times are relative to this reference. The implementation differs by a constant phase $e^{i3\pi/4}$, which cancels from every energy observable. It retains the receiver quadratic phase.

At $f_0 = c/(1\,\mu\text{m})$ the input spectral amplitude is a normalized Gaussian,

$$A(f) = N \exp\left[-\frac{(\pi T_0(f - f_0))^2}{2\log 2}\right], \qquad T_0 = 20\,\text{fs}, \tag{5}$$

$$E(\rho,t) = \int_0^\infty A(f) H(f,\rho) e^{-2\pi i(f-f_0)t}\, df. \tag{6}$$

The positive-frequency normalization differs negligibly from full-line normalization; the numerical band is $f_0 \pm 120\,\text{THz}$ and its omitted full-line spectral energy is 1.4891×10^{-37}. The input intensity has FWHM T_0 and equal-tail 98% width $W_{98} = 39.51636\,\text{fs}$.

For an ideal area-integrating detector (called the bucket), and a coherent passive mode v normalized by $\int_A |v|^2 dA = 1$, the respective observables are

$$P_B(t) = \int_A |E(\rho,t)|^2 dA, \qquad P_v(t) = \left| \int_A v^*(\boldsymbol{\rho}) E(\rho,t) dA \right|^2. \tag{7}$$

These are optical-envelope powers in the chosen normalization. No detector impulse response or noise is included. The mode is fixed across frequency; no spectral equalizer, gain, time-dependent mode or temporal postselection is allowed. The bucket energy $\eta_B = \int P_B dt$ is an efficiency relative to unit emitted energy, not unit energy incident on the solar disk.

3 Collection subject to temporal containment

Let $K = \int |E(t)\rangle\langle E(t)| dt$ and $K_I = \int_I |E(t)\rangle\langle E(t)| dt$ be operators in the receiver's area-weighted spatial space. The quantity of interest is

$$\alpha_*(T) = \sup_{\substack{\|v\|=1,\ |I|=T \\ \langle v, K_I v\rangle \geq 0.98 \langle v, Kv\rangle}} \frac{\langle v, Kv\rangle}{\operatorname{Tr} K}. \tag{8}$$

The interval position is optimized, not fixed at the pulse centre. This is different from the equal-tail width $t_{0.99} - t_{0.01}$ used for descriptive waveforms below. Nonradial mode components collect no signal in this centred monopole model; discarding them and renormalizing cannot decrease collection or change containment. This reduction would not apply to a general nonspherical lens.

For a fixed interval and bucket-normalized K, write $G = K_I - 0.98K$. Any feasible mode satisfies the weak-duality inequality

$$\langle v, Kv\rangle \leq \inf_{\mu \geq 0} \lambda_{\max}(K + \mu G). \tag{9}$$

Explicit feasible vectors provide numerical lower estimates; this dual expression provides upper estimates after allowance for operator errors and interval position. No assertion of general primal–dual equality is needed.

4 Propagation and numerical controls

The saddle is at $b = 7.56661 \times 10^8\,\mathrm{m} = 1.08763R$. The full radial integral is deformed onto a connected vertical segment $r = R - iy$, $0 \le y \le b - R$, followed by $r = b + (1+i)s$, $R - b \le s < \infty$. The integrand is analytic in the right half-plane. With the Abel factor retained, the remote closing arc is quadratically damped. Since $R < b < 2R$, $\mathrm{Re}\, r^2 \ge b(2R - b) > 0$ on the deformed path, allowing removal of the Abel factor by dominated convergence.

On the diagonal, with $u = s/b$, one obtains

$$\frac{\mathrm{Im}\, F}{r_s} = 2u(1+u) - 2\arctan\frac{u}{1+u} \ge 2u^2 \quad (u \ge -1/2). \tag{10}$$

Together with $|J_0(z)| \le e^{|\,\mathrm{Im}\, z|}$, this gives Gaussian tail bounds. The vertical phase has derivative bounded below by

$$d_{\min} = \frac{2r_s R}{R^2 + (b-R)^2} - \frac{R}{Z} > 0, \tag{11}$$

and the source/receiver growth is subtracted before bounding its omitted tail. The supplement supplies the contour bounds, stable logarithm-series evaluation and guard conditions. This is not a ray-arrival histogram or an assumption that only the saddle contributes.

The vertical endpoint contribution is delayed by $[F(R) - F(b)]/c = 131.538\,\mathrm{ns}$ relative to the prompt saddle signal. Its computed energy ratio to the prompt field is $r_e = 2.77723 \times 10^{-10}$. We avoid synthesizing an enormous time window by bounding its effect on each energy operator by

$$\delta_e \le 2\sqrt{r_e} + r_e = 3.33303 \times 10^{-5} \tag{12}$$

in prompt-bucket units. The cross term is retained in this conservative perturbation estimate; the unshifted endpoint field is never simply added to the prompt waveform.

The baseline frontier calculation uses 96 radial, 256 spectral and 128 contour Gauss nodes and 48 time nodes per interval. Nine spatial eigenmodes survive the 10^{-13} relative eigenvalue cutoff. A polynomial approximation across the entire pupil connects the spatial quadrature to the continuum problem. Interval centres in $[-100, 100]\,\mathrm{fs}$ are covered by adaptively subdivided cells. Moving a window changes its operator by its two boundary strips; field and derivative norms bound this change. Windows with centres outside this range are bounded using total bucket energy outside the central interval. The zero-signal complement is included in the perturbed finite-rank dual bound.

The total relative operator allowance is 3.39453×10^{-5}, chiefly the edge term and a 6.05×10^{-7} rank-omission term. It includes an *empirical* 10^{-8} allowance for quadrature and roundoff in both trace and operator quantities. This last allowance has not been proved independently; finite-precision solver slack is also not interval-certified. Therefore the resulting envelopes are numerical estimates, not rigorous total-error certificates, despite analytic bounds on several individual terms.

Three checks address different numerical failure modes. Removing mass and disk recovers composition of free Fresnel propagation with relative errors below 1.4×10^{-14} at $0.8f_0$, f_0 and $1.2f_0$. An opaque-plane, constant-illumination integral agrees with its closed form to 2.4×10^{-13}. Increasing contour order and extent changes selected prompt samples by less than 9×10^{-16} relative to the on-axis field. These controls do not prove uniform error bounds for the physical model.

Stored physical receiver modes were interpolated to 128 radial nodes and recomputed using 384 frequency, 192 contour and 80 time nodes. Energy fractions and containments changed by

less than 2×10^{-14}; this is convergence of the same equations, not an independent propagation implementation. Separately, direct integration of the saved finite spectral sums with the exact window kernel

$$\int_{t_c-T/2}^{t_c+T/2} e^{-2\pi i(f-f')t} dt = T e^{-2\pi i(f-f')t_c} \operatorname{sinc}[(f-f')T] \tag{13}$$

checks temporal integration without time quadrature and agrees within 7×10^{-15} in containment for the original saved witnesses. Here $\operatorname{sinc} x = \sin(\pi x)/(\pi x)$. The supplement permits the same check on the final corrected witnesses.

5 Results

The computed prompt bucket efficiency is $\eta_B = 3.75720914 \times 10^{-12}$. Table 1 gives the qualified numerical envelopes. Lower and upper entries are not confidence intervals and are not measurement uncertainties. They summarize the optimization and stated error allowances.

Window T (fs)	Lower estimate	Upper estimate	Envelope width
30.00000	0.063829	0.100499	0.036670
39.51636	0.500160	0.520159	0.019999
50.00000	0.845302	0.863745	0.018443

Table 1: Estimated maximum collection relative to bucket energy, subject to 98% containment. These are not certified total-error bounds.

The selected envelope-width target was 0.02. The 30 fs calculation stopped after its bounded subdivision allocation and *did not meet that target*. It nevertheless gives a numerical upper estimate far below one-half of bucket collection. The 50 fs calculation contains an explicit mode collecting 0.845365 of bucket energy with nominal containment 0.983857. The large separation between these two durations is the main conditional result. The 39.516 fs lower estimate is only slightly above one-half and should not support a robust threshold claim.

For context, the prompt waveforms of three specified detector choices have the following equal-tail widths, which are not the optimized-window quantity:

Observable	Collected energy	Equal-tail W_{98} (fs)
Area-integrated intensity	3.75721×10^{-12}	51.176
Uniform coherent mode	1.64059×10^{-14}	58.981
Carrier-Bessel-matched mode	3.15929×10^{-12}	46.862

The last mode is proportional to $J_0(k_0 b\rho/z_r)$ and normalized over the aperture; it is fixed across the input band. The contrast between these rows is a concrete demonstration that pupil size and a geometrical ray-delay span do not specify the measured temporal response.

6 Interpretation, limitations and archived status

The calculation predicts a collection cost for demanding short temporal containment within a precisely specified passive receiver class. It does not optimize an arbitrary frequency-dependent optical system, model a fabricated mode converter, infer an achievable data rate, or describe incoherent starlight. Its extremely small absolute collection efficiency follows from the chosen finite coherent transmitter and distances; it is not a universal solar-lens efficiency.

The actual Sun is nonspherical and surrounded by plasma and coronal emission. Solar quadrupole effects may change the spatial interference and invalidate the radial mode reduction;

none were tested here. The atmosphere, alignment, background light, detector bandwidth and noise would also be required for an observational or engineering prediction. Numerical stability of the monopole calculation does not establish those missing physical assumptions.

This research line is parked at the numerical benchmark stage. No instrument data directly verify the specified 650 AU receiver geometry, and this note contains no observational fit or detection. Environmental observations made on different paths could inform future plasma inputs but would not themselves validate this receiver waveform. Any continuation should first establish the closest prior work and independently justify numerical errors, then test nonspherical gravity and plasma. The present publication makes the completed calculation inspectable without claiming those extensions were performed.

7 Conclusion

In the stated scalar model, a fixed coherent receiver mode can retain approximately 85% of bucket energy with a 50 fs, 98%-containment requirement. The estimated maximum falls to approximately 6–10% for 30 fs. This receiver-dependent numerical comparison extends a ray-delay calculation to a finite-band observable while preserving the distinction between geometric delay, received waveform and communications performance.

Reproducibility and usage

The accompanying supplement contains Python code, reference numerical outputs, saved spectral arrays, the precursor certificate scripts, contour derivations, and instructions for reproducing the controls, optimization, refinement and sinc check. Python 3.12.3, NumPy 2.5.0 and SciPy 1.18.0 were used. Reproduction requires no network access or observational dataset. The summary, manuscript and manuscript source use CC BY 4.0; original supplement code and documentation use the MIT licence. Dependencies retain their own licences. Public downloads and corrections are at https://colinocallaghan.com/autonomous-ai-research/solar-lens-waveform-collection/.

References

[1] *Principal-ray delay across a finite solar-lens pupil.* Version 1.0.0, 16 November 2026. Preprint. Published by Colin O'Callaghan. https://colinocallaghan.com/autonomous-ai-research/solar-lens-principal-ray-delay/.