Conditional Timing Exclusions for a Fixed Kerr Orbit in a Stationary Warped Disk: Ansky

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Abstract

Can a fixed orbit crossing a stationary warped disk reproduce the changing recurrence times of Ansky without orbital evolution? We examine a specified timing-only model: an exact bound Kerr geodesic with constant actions, a radial warp, one continuously oriented crossing per polar cycle, fixed published cycle differences, and a leading Rømer–Shapiro arrival-time prescription. The 23 asymmetric timing intervals require departures from any common linear clock to span at least $67385/37673 \simeq 1.78868$ periods. Exact Kerr root identities, bounds on normalized radial cumulative distributions, and a pericenter floor derived from the frequency constraints give a smaller upper bound whenever every observed event satisfies $1 - \mu \geq 10^{-4}$. Here μ is the emitter's direction cosine relative to the specified observer-reference vector. Thus any compatible member must place at least one observed event within approximately 0.81° of far-side conjunction. Stronger necessary alignment conditions hold at lower eccentricity. Certified nonempty orbital families demonstrate that the exclusions are physically populated. A finite search of 105,713 counted generator evaluations found no compatible candidate, but does not establish global infeasibility. The unbounded near-conjunction logarithm and an unresolved connection to a physically selected null-ray arrival leave the full model undecided. These results neither establish orbital expansion nor exclude all geometric interpretations of Ansky.

1 Introduction

A changing interval between successive eruptions need not equal a changing orbital period. If a body crosses a warped disk, the emission phase can move around a fixed orbit; light-travel delays can further change the arrival times. Ansky's measured peak sequence therefore poses a precise inverse problem: can a stationary geometry and a constant-action orbit produce all the assigned event times? The observational timing table and its interpretation are discussed in Ref. [1]. Stationary warped-disk spectral and timing models have also been developed for other quasi-periodic eruption sources[2]. Neither a generic geometric modulation nor an unsuccessful fit settles the specific constrained question considered here.

This paper studies an explicitly defined extension using exact Kerr emitter motion and a specified leading arrival-time formula. It is not an assertion that every component, approximation, prior, or selection rule of a published source model is identical to ours. In particular, the source timing treatment described in Ref. [2] uses a per-orbit approximation, while the inspected timing-generator implementation permits an exact Kerr emitter trajectory with the same closed-form propagation terms[3]. We do not transplant spectral data from other sources, fit eruption energetics, or infer visibility of every disk crossing.

The principal contribution is a conditional analytical exclusion, rather than a best-fit model or a complete no-go theorem. The proof compares a data-imposed minimum timing width with a physical maximum width. It preserves a full polar cycle of crossing-phase freedom and bounds both coordinate-time oscillations and propagation. The final action-domain coverage is obtained

from the original frequency and spin constraints, not an added lower-radius cut. We also report a finite numerical search and nonempty physical examples, keeping these logically distinct from the exclusion theorem.

2 Data and the precise compatibility problem

2.1 Timing intervals and fixed cycle labels

Table 1 gives the complete input. Published epoch n is assigned selected crossing index $j = n + 2$. All consecutive selected crossings $j = 0, \ldots, 32$ must exist, including the ten unobserved labels. The observed set is

$$\mathcal{J} = \{0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 29, 30, 32\}. \tag{1}$$

No new integer aliases or omitted selected cycles are fitted. Both crossing orientations are allowed, but a model must use a single continuously tracked orientation throughout the sequence.

For row i, let $[L_i, U_i] = [t_i - \sigma_{i,-}, t_i + \sigma_{i,+}]$. The tabulated uncertainties already include the adopted 0.1-day peak-model floor. We add neither another floor nor fitted stochastic jitter. The criterion is deterministic interval compatibility, not a simultaneous confidence statement or a goodness-of-fit probability. A successful prediction must have its complete numerical and physical approximation enclosure inside every interval, with total prediction uncertainty at most 0.001 day; a smaller boundary margin requires a correspondingly tighter enclosure. A constant peak delay is absorbed in the common first-event reference, restricted to $[60690.53, 60690.73]$ MJD. Free event-dependent delays are not allowed. The input identifies MJD, but does not establish a more specific time standard.

2.2 Orbit, warp, and parameter domain

Use geometrized units $G = M = c = 1$ for the orbit. The spin is a, distinct from the turning-point semimajor axis $A = (r_a + r_p)/2$; $e = (r_a - r_p)/(r_a + r_p)$ and $p = A(1 - e^2)$. The geodesic has fixed constants (E, L_z, Q), with $0 < E < 1$, $Q \geq 0$, and stable exterior radial libration. Its inclination convention is $x = \cos I = \mathrm{sgn}(L_z)\sqrt{1 - z_-^2}$, where z_-^2 is the maximum value of $\cos^2\theta$; the physical polar limit is treated by the invariant equations rather than division by L_z.

The adopted open ranges are

$$4 < \log_{10}(M/M_\odot) < 8, \qquad 0 < a < 0.998, \tag{2}$$

$$6 < A < 800, \qquad 0 < e < 0.6, \qquad 1^\circ < I < 179^\circ, \tag{3}$$

$$0 < \theta_w < 88^\circ, \qquad 0.05 < r_w < 20, \qquad 0 < i_o < \pi/2. \tag{4}$$

The disk azimuth and the three orbital initial phases are periodic modulo 2π. The exact frequencies from the same orbit must satisfy

$$10 < N_{\mathrm{aps}} := \frac{\Omega_\theta}{\Omega_\theta - \Omega_r} < 100, \qquad 100 < N_{\mathrm{nod}} := \frac{\Omega_\theta}{|\Omega_\theta - \Omega_\phi|} < 30000. \tag{5}$$

The azimuthal frequency is signed; retrograde inclinations are not removed by an extra prior. The analytical exclusion below does not require the nodal cut and therefore holds on a superset of the allowed actions. It neither settles the exact-polar azimuthal convention nor asserts a nonempty exact-polar family.

Let $\hat{\boldsymbol{r}}$ denote the unit direction with Boyer–Lindquist angular coordinates (θ, ϕ). Before a fixed rotation by disk azimuth ϕ_d, the stationary normal is

$$\gamma(r) = 2\sqrt{r_w/r}, \qquad \boldsymbol{\ell}(r) = \left(\sin\theta_w e^{-\gamma}\cos\gamma, \sin\theta_w e^{-\gamma}\sin\gamma, \sqrt{1 - \sin^2\theta_w e^{-2\gamma}}\right). \tag{6}$$

Table 1: All 23 adopted peak times and total asymmetric uncertainties, transcribed from the event table in Ref. [1]. Times and errors are in days. These errors already contain the adopted peak-model floor.

Published epoch n	Selected index j	Peak MJD	σ_-	σ_+
-2	0	60690.63	0.10	0.10
0	2	60709.36	0.10	0.10
1	3	60719.23	0.10	0.10
2	4	60729.32	0.10	0.10
3	5	60739.35	0.10	0.10
4	6	60749.94	0.10	0.10
5	7	60760.46	0.10	0.10
6	8	60770.71	0.10	0.10
7	9	60781.00	0.10	0.10
8	10	60791.46	0.10	0.10
9	11	60802.32	0.10	0.10
10	12	60813.09	0.10	0.10
11	13	60824.27	0.10	0.10
12	14	60835.38	0.11	0.11
13	15	60846.83	0.17	0.24
14	16	60857.78	0.14	0.12
15	17	60869.51	0.10	0.10
16	18	60881.64	0.10	0.10
17	19	60894.43	0.11	0.11
18	20	60906.71	0.12	0.13
27	29	61024.83	1.08	0.10
28	30	61038.33	0.14	0.13
30	32	61067.48	0.22	0.19

A crossing obeys $\boldsymbol{\ell}(r)\cdot\hat{\boldsymbol{r}}=0$. The orientation is the sign of the derivative of this crossing function along the orbit. There must be one selected crossing in each of 33 consecutive, commonly phased polar Mino-time cycles. An arbitrary initial polar phase is allowed: no argument assumes that moving the cycle boundary to a north turning point preserves every possible crossing assignment.

2.3 Arrival prescription and its physical limitation

Fix the observer-reference vector $\boldsymbol{n}=(\sin i_o, 0, \cos i_o)$ and define $\mu=\boldsymbol{n}\cdot\hat{\boldsymbol{r}}$. The physical distant detector convention is $-D\boldsymbol{n}$, with a common far-distance contribution removed. The dimensionless delay is

$$\mathcal{D}(r,\mu)=r\mu-2\log\bigl[r(1-\mu)\bigr]. \tag{7}$$

Thus $\mu\to 1$ is far-side conjunction in this convention. The scalar signs agree with the inspected timing generator at the pinned revision in Ref. [3]; the physical detector placement is our explicit convention, not a claim that the code constructs that detector.

The observed arrival is $T=C+s[t+\mathcal{D}]$, with positive common conversion $s=(1+z)GM/c^3$ expressed in days. Numerical work used the rounded convention $z=0.024$ and $GM_\odot/c^3=4.926580927874239\times10^{-6}$ seconds. This is a specified conversion, not a new precision measurement of either quantity. The analytical width ratio is independent of that common scale.

An acceptable physical witness would additionally need independent exact timelike integration, complete crossing and frequency checks, and a propagation validation against a unique continuously tracked primary null ray. The adopted error allocation requires the source-delay discrepancy to be below 0.0005 day within the total 0.001-day prediction budget, using only one common offset. That is an additional validation condition of this study, not a guarantee derived from the source manuscripts or code. No qualifying candidate was found and no candidate null-propagation certificate was obtained. In particular, "primary" has not been proved equivalent to globally earliest arrival. The exclusion uses the leading prescription as a necessary condition, so it remains valid for any subset that also passes the stronger witness checks.

3 A sharp timing-width obstruction from the data

Suppose arrivals can be written $T_i=C+j_iP+B_i$, $P>0$, with $\operatorname{osc}_i B_i\le\kappa P$. Here osc means maximum minus minimum, not a sinusoidal amplitude. After absorbing the minimum into C, interval compatibility is equivalent, for this relaxation, to

$$L_i-U_k\le(j_i-j_k+\kappa)P\quad\text{for every ordered pair }(i,k). \tag{8}$$

Indeed the possible common intercepts form the intersection of $[L_i-(j_i+\kappa)P,\ U_i-j_iP]$. Nonempty intersection of these one-dimensional intervals is equivalent to all pair inequalities.

Lemma 1 (Exact interval obstruction)**.** *For Table 1, the smallest possible width coefficient in this relaxation is*

$$\kappa_*=\frac{67385}{37673}=1.788681549\ldots,\qquad P_*=\frac{37673}{3200}=11.7728125\text{ days}. \tag{9}$$

Proof. Number table rows from zero. Rows $0, 16, 22$ have selected indices $0, 17, 32$. Two members of Eq. (8), with $q=1/P>0$, give

$$\kappa\ge\max\left\{\frac{3953}{20}q-15,\ 17-\frac{4477}{25}q\right\}. \tag{10}$$

The increasing and decreasing lines meet at $q=3200/37673$ and have common value $67385/37673$, giving a global lower bound. At that period the required residual width is $13477/640$ days. Substitution of the rational decimal table into all $23^2=529$ inequalities verifies that this width suffices. Equivalently, direct maxima of $L_i-j_iP_*$ and minima of $U_i-j_iP_*$ attain precisely that difference. Thus the lower bound is sharp for the relaxation. □

The relaxed intercept is unrestricted and the model's mass limits are not used. Relaxing these restrictions is safe for exclusion. Similarly, using closed data intervals without prediction-error erosion weakens the test: any fully uncertainty-enclosed witness must still pass it. Equation (9) is not a physical orbit fit.

4 From exact Kerr motion to an arrival-width bound

4.1 Separation and the full crossing-phase allowance

Let $\Delta = r^2 - 2r + a^2$, $\Sigma = r^2 + a^2\cos^2\theta$, and $d\tau = \Sigma\, d\lambda$. The separated equations are

$$\mathcal{P}_r = E(r^2 + a^2) - aL_z, \tag{11}$$

$$\mathcal{R}(r) = \mathcal{P}_r^2 - \Delta\{r^2 + (L_z - aE)^2 + Q\}, \tag{12}$$

$$\Theta(\theta) = Q - \cos^2\theta\left[a^2(1 - E^2) + \frac{L_z^2}{\sin^2\theta}\right], \tag{13}$$

$$(r')^2 = \mathcal{R}(r), \qquad (\theta')^2 = \Theta(\theta), \tag{14}$$

$$t' = T_r + T_\theta, \quad T_r = \frac{(r^2 + a^2)\mathcal{P}_r}{\Delta}, \quad T_\theta = aL_z - a^2E\sin^2\theta. \tag{15}$$

Primes denote Mino derivatives. Write the full radial and polar periods as $\Lambda_r, \Lambda_\theta$, set $\Upsilon_i = 2\pi/\Lambda_i$, and define

$$\Gamma = \langle T_r\rangle + \langle T_\theta\rangle, \qquad \Omega_i = \Upsilon_i/\Gamma. \tag{16}$$

The brackets denote the corresponding periodic Mino averages. Exactly,

$$t(\lambda) = C_t + \Gamma\lambda + F_r(\lambda) + F_\theta(\lambda), \quad F_r' = T_r - \langle T_r\rangle, \quad F_\theta' = T_\theta - \langle T_\theta\rangle. \tag{17}$$

The functions F_r, F_θ are periodic. In particular, coordinate-time polar cycles need not have identical duration: the radial primitive changes between them.

The crossing contract implies $\lambda_j = \lambda_* + j\Lambda_\theta + \delta_j$, $0 \le \delta_j \le \Lambda_\theta$. Consequently, with $P = s\Gamma\Lambda_\theta$,

$$\frac{\operatorname{osc}_{j\in\mathcal{J}}(T_j - jP)}{P} \le 1 + \frac{W_r + W_\theta + W_{\mathcal{D}}}{\Gamma\Lambda_\theta}, \tag{18}$$

where W_r, W_θ bound full primitive widths and $W_{\mathcal{D}}$ bounds delay width at observed crossings. The term 1 retains a *full* polar cycle. No thin-warp phase-window assumption or phase-gauge relabeling is needed. Dropping root-orientation correlations makes this a necessary condition on a larger set, which is suitable for exclusion.

4.2 A positive-density bound

For a periodic function f of period Λ, mean m, and bounds $\ell \le f \le u$, its centered primitive satisfies

$$\operatorname{osc} F \le \Lambda\frac{(u - m)(m - \ell)}{u - \ell} \le \frac{\Lambda(u - \ell)}{4}. \tag{19}$$

To prove the first inequality, an arc of length h between primitive extrema has integral at most both $(u - m)h$ and $(m - \ell)(\Lambda - h)$. Maximizing the smaller expression over h yields the result. The constant case follows by continuity. For $\ell > 0$, optimization over the mean gives

$$\frac{\operatorname{osc} F}{m\Lambda} \le v(u/\ell), \qquad v(K) = \frac{\sqrt{K} - 1}{\sqrt{K} + 1}. \tag{20}$$

The same proof applies to any positive probability base measure reweighted by a positive function with maximum/minimum ratio at most K: its total variation distance from the normalized reweighting is at most $v(K)$. Total variation here is half the L^1 distance. This convention avoids an extra factor of two.

4.3 Radial time density and all angular-momentum signs

Define $c_0 = a^2 - aL_z/E$ and

$$q_r(r) = \frac{T_r}{Er^2} = \frac{(r^2+a^2)(r^2+c_0)}{r^2\Delta}. \tag{21}$$

Every orbit initially has $r \geq r_p = A(1-e) > 12/5 > 2$. Future-directed exterior motion gives $\mathcal{P}_r > 0$. One way to see the sign is to contract the four-velocity with the future timelike Carter vector $(r^2+a^2)\partial_t + a\partial_\phi$. Since $\mathcal{R} \geq 0$, $\mathcal{P}_r > \sqrt{\Delta}\,|L_z - aE|$. Algebra then gives

$$-\frac{ar^2}{\sqrt{\Delta}+a} < c_0 < \frac{ar^2}{\sqrt{\Delta}-a}. \tag{22}$$

Both denominators are positive. The identity $\Delta - a^2(r-1)^2 = (1-a^2)r(r-2) \geq 0$ implies $c_0 > -r$. Differentiation yields $q_r' = 2N/(r^3\Delta^2)$, with

$$N = -r^5 - c_0 r^4 + (a^2+c_0)r^3 - 2a^2c_0r^2 + 3a^2c_0r - a^4c_0. \tag{23}$$

The coefficient of c_0 is negative for $r > 2$. Inserting its lower bound gives $N \leq -r[r^3 - 3a^2r^2 + 3a^2r - a^4] < 0$: the bracket decreases with $a^2 \in [0,1]$ and at $a^2 = 1$ is $(r-1)^3$. Thus q_r decreases, without any need to assert monotonicity of T_r itself.

Equation (22) also gives

$$1 < q_r(r) < \frac{r_p^2+1}{(r_p-1)(r_p-2)} =: Q_p, \qquad \frac{\max(c_0,0)}{r_p^2} < \frac{1}{r_p-2}. \tag{24}$$

For completeness, bound q_r between $(r^2+a^2)/[\sqrt{\Delta}(\sqrt{\Delta}+a)]$ and $(r^2+a^2)/[\sqrt{\Delta}(\sqrt{\Delta}-a)]$. The lower expression exceeds one, using $2r > a\sqrt{\Delta}$; the upper increases with a and at $a = 1$ becomes $(r^2+1)/[(r-1)(r-2)]$, which decreases with $r > 2$. The second inequality in Eq. (24) follows by maximizing $a/(\sqrt{\Delta}-a)$ at $a = 1$. For $r_p \geq R$ and $r_a/r_p \leq \rho$, these facts imply

$$\frac{\sup T_r}{\inf T_r} \leq \max\{\rho^2, Q_R\}, \qquad d_{\text{den}} := v\Big(\max\{\rho^2, Q_R\}\Big), \quad Q_R = \frac{R^2+1}{(R-1)(R-2)}. \tag{25}$$

If the numerator radius exceeds the denominator radius, monotone q_r bounds the ratio by the squared radius ratio; in the opposite ordering, $q_r > 1$ and its upper bound suffice. This explains why monotone T_r is unnecessary.

5 Frequency constraints, inner roots, and a derived action floor

Write the radial roots as $u = r_a > v = r_p > s_3 = r_3 \geq t_4 = r_4 \geq 0$ and $\epsilon = 1 - E^2 > 0$. For stable exterior libration,

$$\mathcal{R}(r) = \epsilon(u-r)(r-v)(r-s_3)(r-t_4). \tag{26}$$

The inner-root ordering follows from the quartic signs at zero, the horizon, the forbidden interval below pericenter, and the allowed outer interval. Vanishing inner roots in limiting cases are handled continuously; a radial separatrix is not included. Let

$$S_2 = uv + us_3 + ut_4 + vs_3 + vt_4 + s_3t_4, \qquad J^2 = L_z^2 + Q. \tag{27}$$

Coefficient comparison gives $\epsilon S_2 = J^2 + a^2\epsilon$ and $\epsilon uvs_3t_4 = a^2Q$. Writing $z = \cos\theta$ gives $(z')^2 = Q - (J^2 + a^2\epsilon)z^2 + a^2\epsilon z^4$. The larger frequency-root scale h^2 therefore satisfies $(h^2)^2 - (J^2+a^2\epsilon)h^2 + a^2\epsilon Q = 0$, so $H = h^2/\epsilon$ is the larger root of

$$H^2 - S_2H + uvs_3t_4 = 0. \tag{28}$$

Put $C_r = (u - s_3)(v - t_4)$, $m_r = (u - v)(s_3 - t_4)/C_r$, and $m_\theta = uvs_3t_4/H^2$. With $K(m)$ the complete elliptic integral in parameter convention,

$$\Upsilon_r = \frac{\pi\sqrt{\epsilon C_r}}{2K(m_r)}, \qquad \Upsilon_\theta = \frac{\pi\sqrt{\epsilon H}}{2K(m_\theta)}. \tag{29}$$

The elementary bounds $\pi/2 \le K(m) \le \pi/[2\sqrt{1-m}]$ imply

$$\left(\frac{\Upsilon_r}{\Upsilon_\theta}\right)^2 \le \frac{C_r}{\sqrt{S_2^2 - 4uvs_3t_4}}. \tag{30}$$

No assumption that $K(m_\theta) \le K(m_r)$ is made.

Since

$$S_2^2 - 4uvs_3t_4 - (S_2 - 2s_3t_4)^2 = 4s_3t_4(u+v)(s_3+t_4) \ge 0, \tag{31}$$

the denominator in Eq. (30) is at least $uv + (u+v)s_3$. Write $k = v/u = (1-e)/(1+e)$. The apsidal cut gives $\Upsilon_r/\Upsilon_\theta > 9/10$, so

$$\frac{81}{100} < \frac{1 - k(s_3/v)}{1 + (1+k)(s_3/v)}, \qquad 0 \le \frac{t_4}{v} \le \frac{s_3}{v} < y(k) := \frac{19}{81 + 181k}. \tag{32}$$

Lemma 2 (Pericenter floor implied by the original cuts)**.** *Every orbit in the adopted action domain satisfies $r_p > 8585/1216 > 7$. For $e \in [e_-, e_+]$, a sharper uniform floor is*

$$r_p > R := \frac{17}{16y(k_-)}, \qquad k_- := \frac{1-e_+}{1+e_+}. \tag{33}$$

Proof. The horizon $r_+ = 1 + \sqrt{1-a^2}$ lies below $v > 12/5$. Because $\mathcal{R} < 0$ on (s_3, v) but $\mathcal{R}(r_+) = \mathcal{P}_r(r_+)^2 \ge 0$, it cannot lie strictly in that interval. Hence $r_+ \le s_3$, allowing equality at a horizon root. The spin bound $a < 499/500$ gives $r_+ > 17/16$, since $1 - (499/500)^2 > 1/256$. Equation (32) yields $v > 17/(16y(k_-))$. Finally $e_+ \le 3/5$ gives $y(k_-) \le 76/505$. □

This floor is a consequence of the model; it removes no permitted action by an additional prior.

5.1 Energy, mean radius, and polar frequency

Coefficient matching also gives $\epsilon = 2/(u+v+s_3+t_4)$. Thus

$$E^2 \ge 1 - \frac{1-e}{v} \ge 1 - \frac{1-e_-}{R}. \tag{34}$$

Let $\rho_+ = (1+e_+)/(1-e_+)$ and $y_+ = y(k_-)$. Since $K(m_\theta) \ge \pi/2$ and $H \le S_2$,

$$\Upsilon_\theta^2 \le \epsilon S_2 \le C_b v, \qquad C_b = 2\frac{\rho_+ + 2(\rho_+ + 1)y_+ + y_+^2}{\rho_+ + 1 + 2y_+}. \tag{35}$$

The rational function $F(\rho, A_3, A_4) = [\rho + (\rho+1)(A_3+A_4) + A_3A_4]/[\rho + 1 + A_3 + A_4]$ increases in all three arguments for nonnegative A_3, A_4. Up to its positive squared denominator, the derivative in A_3 is $\rho^2 + \rho + 1 + (\rho+1)A_4 + A_4^2$, and the derivative in ρ is $1 + A_3 + A_4 + A_3^2 + A_3A_4 + A_4^2$; the third follows by symmetry. Substituting their upper bounds proves Eq. (35).

On the outward radial half-cycle use $r = p/(1 + e\cos\chi)$, $0 \le \chi \le \pi$. Cancellation of the outer turning-point factors gives a Mino density proportional to

$$w(\chi) = \left(1 - \frac{s_3}{r}\right)^{-1/2} \left(1 - \frac{t_4}{r}\right)^{-1/2}. \tag{36}$$

It decreases with r. Applying Eq. (32) to its extremal ratio yields

$$\frac{w_{\min}}{w_{\max}} \geq W(k) := \frac{62+181k}{81+162k} \geq W(k_-) =: W_b. \tag{37}$$

Indeed $(1-y)/(1-ky)$ decreases with y, and $W(k)$ increases with k. The elementary integral of r^2 under uniform χ is $\pi p^2/(1-e^2)^{3/2}$. Therefore

$$\frac{\langle r^2\rangle}{v^2} \geq B := \max\left\{1,\ W_b\sqrt{\frac{1+e_-}{(1-e_-)^3}}\right\}. \tag{38}$$

The lower eccentricity endpoint is essential in this lower mean bound.

Because $T_\theta = E(a^2\cos^2\theta - c_0)$, Eqs. (24) and (38) imply, for $G_b := B - 1/(R-2) > 0$,

$$\Gamma > Ev^2 G_b, \qquad \frac{\langle T_r\rangle}{\Gamma} < \frac{B}{G_b}. \tag{39}$$

For the second inequality use $\langle T_r\rangle > EBv^2$ and $\Gamma \geq \langle T_r\rangle - E\max(c_0, 0)$. These bounds retain both signs of L_z. They assign different factors to the radial primitive (B/G_b) and the polar and delay terms ($1/G_b$).

6 A sharper radial bound from normalized cumulative distributions

The density bound in Eq. (25) ignores where radial motion spends time. A sharper comparison uses two probability cumulative distribution functions on the *same outward half-cycle*:

$$G_r(\chi) = \frac{\int_0^\chi w\, d\chi'}{\int_0^\pi w\, d\chi'}, \qquad H_r(\chi) = \frac{\int_0^\chi T_r w\, d\chi'}{\int_0^\pi T_r w\, d\chi'}. \tag{40}$$

If F_r vanishes at pericenter, then on this half-cycle $F_r = (\langle T_r\rangle\Lambda_r/2)(H_r - G_r)$. Reflection of the radial libration gives $F_r(\Lambda_r - \lambda) = -F_r(\lambda)$ on the corresponding inward phase. The full width is consequently

$$\frac{W_r}{\langle T_r\rangle\Lambda_r} = \sup_{0\leq\chi\leq\pi} |H_r - G_r|. \tag{41}$$

The reflection factor two cancels the half-cycle factor $1/2$.

Let $G_0 = \chi/\pi$ and $H_0 = M_K(\chi)/\pi$, where the Kepler eccentric anomaly E_K and mean anomaly M_K obey

$$\tan(E_K/2) = \sqrt{\frac{1-e}{1+e}}\tan(\chi/2), \qquad M_K = E_K - e\sin E_K. \tag{42}$$

The density of H_0 is proportional to r^2, and $H_0 \leq G_0$. This is a reference measure, not a Newtonian replacement for the Kerr trajectory. The exact G_r is the normalized reweighting of G_0 by w, while H_r is the normalized reweighting of H_0 by $q_r w$. Both weights decrease with χ by Section 4.3 and Eq. (36).

For any positive base probability measure, reweighting by a decreasing positive weight increases its cumulative distribution. At a cut with base probability p, the difference is $p(1-p)(\overline{w}_{\text{prefix}} - \overline{w}_{\text{suffix}})/\overline{w} \geq 0$. Combining this order with Eq. (20) gives

$$0 \leq G_r - G_0 \leq \varepsilon_G := v(1/W_b), \qquad 0 \leq H_r - H_0 \leq \varepsilon_H := v(Q_R/W_b). \tag{43}$$

If $D_K = \sup(G_0 - H_0)$, the two directed differences satisfy

$$G_r - H_r \leq D_K + \varepsilon_G, \qquad H_r - G_r \leq \varepsilon_H. \tag{44}$$

Therefore

$$d_{\mathrm{CDF}} := \min\{d_{\mathrm{den}},\ \max(D_K^{\mathrm{up}} + \varepsilon_G, \varepsilon_H)\} \tag{45}$$

is an upper bound on Eq. (41). Adding both total-variation errors would be valid but unnecessarily loose; no assumption that $H_r \le G_r$ is required.

A simple explicit Kepler bound suffices. Put $q_K = e/(1+\sqrt{1-e^2})$. The tangent half-angle relation gives $\max(\chi - E_K) = 2\arcsin q_K$: maximizing $(s_K - 1)t/(1 + s_K t^2)$ with $s_K = \sqrt{(1+e)/(1-e)}$ and $t = \tan(E_K/2)$ proves this identity. It follows that

$$D_K \le \frac{2\arcsin q_K + e}{\pi} \le \frac{2q_K/\sqrt{1-q_K^2} + e}{\pi}. \tag{46}$$

The second inequality integrates the increasing derivative of arcsin. The right side increases with e; its outward upper bound at e_+ is D_K^{up}. Adding the separate maxima of $\chi - E_K$ and $e\sin E_K$ is conservative, without asserting that they occur together.

7 Propagation, geometry, and the finite exact certificate

7.1 Delay-width bound

Suppose $|\mu| \le m \le 1$ and $1 - \mu \ge \delta > 0$ at the observed crossings. Since $v \le r \le u \le \rho_+ v$,

$$W_{\mathcal{D}} \le 2\rho_+ m v + 2\log\left(\frac{\rho_+(1+m)}{\delta}\right) \le 2\rho_+ m v + \frac{7}{5}k_{\log}, \tag{47}$$

where $k_{\log}$ is the smallest nonnegative integer for which $\rho_+(1+m)/\delta \le 2^{k_{\log}}$. The inequality $\log 2 < 7/10$ follows because the order-four positive Taylor sum for $\exp(7/10)$ already exceeds two. The radial logarithm is included, not just angular delay.

Using Eqs. (34), (35), and $2\pi > 157/25$, define

$$D_b = \frac{\sqrt{C_b}}{(157/25)\sqrt{1-(1-e_-)/R}}\left[\frac{2\rho_+ m}{\sqrt{R}} + \frac{7k_{\log}/5}{R\sqrt{R}}\right]. \tag{48}$$

Then $W_{\mathcal{D}}/(\Gamma\Lambda_\theta) \le D_b/G_b$. Replacing v by its derived lower bound R is conservative because each power of v in this upper expression decreases. The polar primitive obeys Eq. (19), with $\mathrm{range}(T_\theta) \le a^2 E$, giving $W_\theta/(\Gamma\Lambda_\theta) \le 1/(4R^2 G_b)$. Finally Eq. (5) implies $\Lambda_r/\Lambda_\theta < 10/9$. Substitution into Eq. (18) yields the central sufficient exclusion coefficient:

$$\boxed{K_b = 1 + \frac{10}{9}d_{\mathrm{CDF}}\frac{B}{G_b} + \frac{1/(4R^2) + D_b}{G_b}}. \tag{49}$$

Whenever $K_b < \kappa_*$, no member of that action band and geometry condition fits Table 1.

The clearance requirement in this argument concerns only the observed set $\mathcal{J}$: only those residuals enter the data envelope. The coordinate-time primitive bounds are global, and all 33 crossings remain subject to the original cycle, orientation, continuity, and physical requirements. No unobserved crossing is deleted from a prospective witness. Earlier, auxiliary radius-threshold maps used a stronger all-selected-crossing clearance antecedent; their interpretation is kept separate below.

7.2 Geometric ways to guarantee clearance

At a crossing, $\boldsymbol{\ell}\cdot\hat{\boldsymbol{r}} = 0$. Projection onto its disk plane gives

$$|\mu| \le \sqrt{1 - (\boldsymbol{n}\cdot\boldsymbol{\ell})^2}. \tag{50}$$

Thus $|\boldsymbol{n}\cdot\boldsymbol{\ell}| \geq \eta > 0$ implies $1-\mu \geq \eta^2/2$, since $1-\sqrt{1-\eta^2} \geq \eta^2/2$. We use the looser $m=1$ throughout the final finite map.

For all allowed warps, $r_a < 1280$ and $r_w > 0.05$ imply $\gamma > 1/80$ and $\sin\beta = \sin\theta_w e^{-\gamma} < 80/81$; hence $\cos\beta > \sqrt{161}/81$. An observer cone $i_o \leq i_{\max}$ has the conservative bound

$$\boldsymbol{n}\cdot\boldsymbol{\ell} \geq \eta_{\text{all}} := (1 - i_{\max}^2/2)\frac{\sqrt{161}}{81} - i_{\max}\frac{80}{81}. \tag{51}$$

The tested cones have positive η_{all}. Alternatively, a tilt restriction $\theta_w \leq b$ gives $\beta \leq b$ and, if $(i_{\max}+b)^2 < 2$,

$$\boldsymbol{n}\cdot\boldsymbol{\ell} \geq \cos(i_{\max}+b) \geq 1 - \frac{(i_{\max}+b)^2}{2} =: \eta_b > 0. \tag{52}$$

These are uniform in disk azimuth, orbital phases, and allowed warp radii. They are derived clearance bounds, not additional photon occultation rules.

7.3 Finite evaluation with outward rational bounds

The continuum action domain is covered by the 60 closed rational bands $[j/100, (j+1)/100]$, $j = 0, \ldots, 59$, intersected with the original open interval $0 < e < 0.6$. For each band, derive R from Eq. (33) and evaluate Eq. (49). No independent pericenter threshold is imposed. The 21 geometric conditions comprise:

1. five all-warp observer cones $i_o \leq 0.01, 0.05, 0.10, 0.125, 0.15$ radians, using Eq. (51);
2. six observed-event clearances $\delta = 10^{-1}, 10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}, 10^{-32}$;
3. the ten pairs $b \in \{0.1, 0.3, 0.5\}$, $i_{\max} \in \{0.3, 0.6, 0.9, 1.2\}$ satisfying $(b+i_{\max})^2 < 2$, using Eq. (52).

All angles here are radians. This gives 1260 sufficient-bound rows. A geometric condition is excluded over the *entire* action domain only if all of its 60 rows pass. Counts across overlapping conditions are not volume fractions.

Every input decimal is converted directly to a rational number. For a rational $x \geq 0$, integer square root supplies

$$\frac{n}{10^{24}} \leq \sqrt{x} \leq \frac{n+1}{10^{24}}, \qquad n = \left\lfloor \sqrt{\lfloor 10^{48} x \rfloor} \right\rfloor, \tag{53}$$

with squared endpoints checked. Lower endpoints are used for B, energy and square-root denominators, and upper endpoints for numerator and error bounds, following each expression's monotonicity. Strict comparison with 67385/37673 is rational. The lower bound $\pi > 157/50$ can be certified without a floating-point transcendental library: Machin's identity and alternating arctangent series give

$$\pi = 16\arctan(1/5) - 4\arctan(1/239) > 16\left(\frac{1}{5} - \frac{1}{3\cdot 5^3}\right) - \frac{4}{239} > \frac{157}{50}. \tag{54}$$

The tangent addition formula verifies the identity in its principal range. These elementary enclosures, the logarithm comparison, and the data pairs are sufficient to reproduce the final scalar certificate; no orbit sampling is involved in it.

8 Results

8.1 Conditional exclusion over the complete action domain

Theorem 1 (Necessary observed-event conjunction)**.** *For the model, fixed cycle labels, and intervals of Section 2, any compatible member has at least one observed crossing $j \in \mathcal{J}$ for which*

$$1 - \mu_j < 10^{-4}. \tag{55}$$

All allowed actions satisfying $1-\mu_j \geq 10^{-4}$ *at every observed event are excluded, including all angular-momentum signs and regular physical polar configurations subject to the stated assumptions.*

Proof. Evaluate Eq. (49) on the 60 bands with $m = 1$, $\delta = 10^{-4}$ and the derived floors of Lemma 2. Every rational strict gap $\kappa_* - K_b$ is positive. The largest outward upper coefficient is $1.7735738287515146\ldots$, below $1.7886815491200594\ldots$. The rational row values, endpoints, and positive gaps are supplied in the reproducibility material. The bands cover every $0 < e < 0.6$, and all preceding inequalities hold throughout each band, so Lemma 1 excludes every member under the stated clearance. Taking the contrapositive proves the claim. The argument uses no division by L_z, no azimuthal frequency, and no numerical backend evaluation; regular physical polar motion with finite positive polar period is included before the additional nodal cut is imposed. □

Since μ is a direction cosine, Eq. (55) corresponds to separation from $\boldsymbol{n}$ below $\arccos(1-10^{-4}) \simeq 0.81^\circ$. This is far-side conjunction under the detector convention in Section 2.3. It is necessary, not sufficient, and carries no probability or physical-volume interpretation.

The smaller-clearance families give the stronger eccentricity-dependent necessities in Table 2. All 60 bands pass for $\delta = 0.1, 0.01, 10^{-4}$. The $10^{-8}, 10^{-16}, 10^{-32}$ families have, respectively, 6, 22, and 50 failing sufficient-bound rows. The passing closed bands cover the interior endpoints in the table; source endpoints zero and 0.6 remain excluded.

Table 2: Necessary alignment at at least one of the 23 observed events. These are conditions on any compatible model, not fitted orbits.

Eccentricity	Necessary observed-event condition
$0 < e \leq 0.10$	$1-\mu < 10^{-32}$
$0.10 < e \leq 0.38$	$1-\mu < 10^{-16}$
$0.38 < e \leq 0.54$	$1-\mu < 10^{-8}$
$0.54 < e < 0.60$	$1-\mu < 10^{-4}$

All five all-warp cones and all ten bounded-warp angle conditions pass every band. Their maximal declared conditions reduce by containment to

$$\begin{aligned} &i_o \leq 0.15 \quad \text{for all allowed warp tilts,} \\ &\theta_w \leq 0.10, \quad i_o \leq 1.20, \\ &\theta_w \leq 0.50, \quad i_o \leq 0.90. \end{aligned} \tag{56}$$

Each excludes every allowed action and phase under that geometry. The worst coefficient for the largest all-warp cone is $1.782323088528519\ldots < \kappa_*$. In total 1182 of the 1260 action-map rows pass, and 18 of the 21 geometric conditions pass all 60 bands. A failed sufficient inequality does not establish that an orbit fitting the data exists.

8.2 Nonempty examples and auxiliary radius-threshold certificates

The exclusions are not supported only by possibly empty parameter descriptions. Interval calculations establish nonzero-width constants-defined families with four ordered simple radial roots, admissible polar motion, strict frequency cuts, and a uniformly transverse emission sequence for $0 < \theta_w < 0.005$ and $0 < i_o < 0.01$ radians, over all allowed warp radii. Two examples have

$$e \in [0.56991727, 0.56991759], \qquad r_p \in [63.70877, 63.70881], \tag{57}$$

$$e \in [0.5698152255, 0.5698159246], \qquad r_p \in [38.23603820, 38.23604108]. \tag{58}$$

Displayed decimal enclosures are explanatory outward roundings; the exact rational boxes and inequalities are in the supplement. For the second family, the frequency-ratio enclosures are $N_{\rm aps} \in [16.40628, 24.33868]$ and $N_{\rm nod} \in [2903.76, 11176.82]$, strictly within the adopted cuts.

One exact description of the second constants box is

$$a \in [49999, 50001]/10^6, \tag{59}$$

$$E^2 \in [16813501998299747, 16813502001700253]/17002530000000000, \tag{60}$$

$$L_z^2 \in [299999999811083, 300000000188917]/18891700000000, \quad L_z > 0, \tag{61}$$

$$Q \in [899999999811083, 900000000188917]/18891700000000. \tag{62}$$

Radial root signs and disjoint brackets, polar-root bounds, elliptic-integral bounds, and nodal integrand enclosures certify every member, not just the central point. At a normalized crossing $F = \boldsymbol{\ell} \cdot \widehat{\boldsymbol{r}} = 0$, with $b = \sin\beta$, $c_\beta = \cos\beta$, $z = \cos\theta$, and $\psi = \phi - \gamma - \phi_d$,

$$F_\theta = -\frac{c_\beta}{\sin\theta}, \quad F_\phi = -b\sin\theta\sin\psi, \quad F_r = -\frac{\gamma}{2r}\left(\frac{z}{c_\beta} + b\sin\theta\sin\psi\right). \tag{63}$$

The certified dominance of $|F_\theta\theta'|$ over $|F_\phi\phi'| + |F_r r'|$ fixes the root orientation. Opposite signs at the polar turning points ensure existence, and fixed orientation at every possible root ensures uniqueness on each polar half-cycle. Choosing the initial cycle boundary at a north turning point supplies a valid emission sequence for these examples; it is not assumed equivalent for all possible models. These boxes certify physical actions and emission crossings, not a timing fit, arrival ordering for all events, or null-propagation accuracy.

A separate 1764-row map uses the same CDF method with explicit pericenter thresholds

$$R \in \{10, 12, 15, 20, 30, 40, 50, 75, 100, 150, 200, 300\},$$

seven bands with boundaries $0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.55, 0.6$, and the same 21 geometry conditions. It retains the older all-33-selected-events clearance antecedent. Its delay scale uses the weaker but valid estimates $E \geq \sqrt{1 - 2/R}$ and $\Upsilon_\theta \leq \sqrt{4r_p + 1}$. Of these rows, 1661 yield strict exclusions. The midradius family lies inside 75 passing rows, and outside all previously obtained coarse radius/eccentricity exclusions and earlier small low-eccentricity boxes. This is one populated incremental excluded family, not 75 separate regions. The final source-action theorem no longer needs the explicit R thresholds. Neither row counts nor overlapping interval boxes define a measure of coverage in physical parameter space.

8.3 Finite numerical search and its limits

The numerical work complements the proof by seeking a counterexample. Across the full investigation 105,713 generator evaluations were counted: 8,353 earlier exploratory calls, followed by 97,336 broader search calls and 24 numerical controls. The broader search sampled the full specified coordinate ranges, both oriented crossing signs, and physically informed and boundary strata. Exact Kerr trajectories were supplied by KerrGeoPy; continuous scalar root solving refined sampled crossing brackets. Sixteen local basins used feasible-incumbent pattern searches, so invalid geometry could not replace an existing valid point. Grid refinement and paired-versus-sequential checks tested numerical consistency; agreement of grids was never treated as proof of complete root topology.

For each saved arrival shape, the common mass scale and reference epoch were optimized by linear programming. One diagnostic minimizes an expansion factor $\rho_{\rm fit}$ multiplying each asymmetric observational error. Compatibility would require $\rho_{\rm fit} \leq 1$ before the additional uncertainty and physical checks. The best recorded factor was 78.24298880565333. Separately maximizing the absolute interval margin for that *same saved shape* gives -10.842982047 days.

These are different objectives: the stored factor-optimal solution has a different margin, and neither quantity is a global optimum over all physical shapes.

The broader calls comprise 53,247 prediction records, 38,631 frequency-filter outcomes, 1,148 sampled crossing-branch filter outcomes, and 4,310 backend or numerical failures. The latter split into 2,256 unstable-orbit backend messages, 1,340 bracketing errors, and 714 nonfinite-value errors; none is counted as an analytical physical exclusion. Thirteen local basins reached their finite evaluation limits and three stopped at finite-stencil resolution. No stationarity certificate follows. No point satisfied the original intervals, no candidate entered full independent physical validation, and no certified timing witness resulted.

Finite sampling, including both signs and every coordinate range, does not cover a continuum. Nor do sampled crossing filters rigorously reject unsampled crossings or near-tangencies. The analytic theorem covers regular physical polar configurations under its own assumptions even where the numerical backend failed, because its invariant proof does not depend on that backend. Outside the proved geometry, numerical failures remain unresolved. The numerical evidence does not replace the conditional theorem and does not turn its complement into an excluded set.

9 Interpretation and unresolved physics

The theorem restricts *where a successful model would have to operate*. It does not prove such a model exists. Close alignment could produce a large individual delay, but the required 23 delays must arise from one orbit, one stationary warp, one orientation, and the fixed cycle assignments. Their correlations are not captured by independent width bounds.

Conversely, the remaining logarithmic freedom cannot be dismissed by another finite grid. For the defined prescription, $-2\log[r(1-\mu)] \to +\infty$ as $\mu \to 1$. Removing the undefined point $\mu = 1$ does not yield a finite neighborhood bound. Imposing a conjunction cut, occultation condition, or finite lensing approximation without justification would change the question. The stronger low-eccentricity requirements in Table 2 emphasize this limitation; they are not evidence that the leading approximation remains accurate at those alignments.

The inspected manuscript timing sections and the pinned timing/trajectory code[1–3] do not establish a selection theorem equating a continuously tracked primary null ray with the globally earliest arrival at a fixed observer. Nor do they supply the uniform exact-null error bound needed here near conjunction. This statement is limited to those timing prescriptions and inspected modules, not every uninspected dependency or publication. A comparison causal path bounds earliest arrival, but need not bound an arbitrarily continued image branch. Substituting earliest arrival into the current model without that missing implication would be a different problem.

A complete negative answer would need a certified coupled-event exclusion of the residual conjunction geometry, or a justified optical bound connected to the same arrival prescription and error contract. A positive answer requires a candidate satisfying all fixed physical and cycle conditions, followed by independent trajectory, crossing, and propagation enclosures inside all 23 intervals. Neither is supplied by the present evidence. The exclusions also do not assess joint spectral-energy distributions, eruption fluences or durations, observer selection, or the actual cause of Ansky's eruptions. They do not establish orbital expansion and do not refute every published warped-disk interpretation.

10 Conclusion

The Ansky timing intervals impose a sharp minimum departure width of 67385/37673 periods from any common linear clock. Exact Kerr bounds, retaining the full crossing-phase allowance, exclude the specified stationary, fixed-action model whenever all observed events remain at clearance $1 - \mu \geq 10^{-4}$. This conclusion spans the adopted action domain because the necessary pericenter floor follows from the original spin and frequency constraints. Lower eccentricities require still closer

observed-event alignment. Nonempty physical examples support the subdomain interpretation, while a substantial finite search found no compatible candidate. The full model remains unresolved because the defined logarithmic propagation delay is unbounded near conjunction and no valid global replacement bound has been established.

Data and reproducibility

The event table, exact rational scalar certificates, physical-box enclosures, and programs reproducing those analytical checks accompany version 1.0.0. A reader can reproduce the interval obstruction from Table 1, reconstruct Eqs. (33)–(49) on the declared bands using integer and rational arithmetic, and verify the strict gaps and physical-box certificates with the documented commands. The supplement does not contain the exploratory orbital-search records or an environment for rerunning that search; the search diagnostics in Section 8.3 are reported results, not results reproducible from this supplement. The analytical exclusions do not rely on those search diagnostics. Source code for this manuscript is provided with the release.
Article page and version information.
Reproducibility supplement, version 1.0.0.

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